Indian Institute of Technology Madras Zanzibar
MTech of Data Science and AI

Technical Aptitude

1. Consider a system of two linear equations in two variables x and y. What does it mean when this system of linear equations has infinitely many solutions?

   A. The straight lines corresponding to the two equations have different slopes and intersect at one point.

   B. The two equations represent the same line.

   C. The straight lines corresponding to the two equations are parallel but have different y-intercepts.

   D. The system has no solution.

   [MCQ - 1 mark]
   Correct Answer: B

2. What do the **singular values** in the singular value decomposition (SVD) of a matrix X represent?

   A. The diagonal entries of X

   B. The square roots of the eigenvalues of X

   C. The eigenvalues of X

   D. The square roots of the eigenvalues of $X^T X$

   [MCQ - 1 mark]
   Correct Answer: D

3. What is the dimension of the vector space of all symmetric 2×2 real matrices with trace equal to zero?

   A. 4

   B. 3

   C. 2

   D. 1

   [MCQ - 1 Mark]

   Correct Answer: C

4. Let $T : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be a linear transformation such that $T(A) = 0$ whenever A is symmetric or skew-symmetric. Then the rank of T is:

   A. $\frac{n(n+1)}{2}$

   B. $\frac{n(n-1)}{2}$

   C. n

   D. 0

   [MCQ - 1 Mark]
   Correct Answer: D

5. Which of the following are eigenvalues of the matrix

$$X = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

A. 2

B. -2

C. 4

D. -4

[MSQ - 1 Mark]

Answer: A and B.

6. What is the Laplace transform of $f(t) = t^3, t \geq 0$?

A. $\frac{1}{s^4}$

B. $\frac{3}{s^4}$

C. $\frac{6}{s^4}$

D. $\frac{6}{s^3}$

[MCQ - 1 mark]
Correct Answer: C

7. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$f(a, b) = (2a + b,\ 4a + 2b).$$

Then:

A. f is onto but not one-to-one.

B. f is one-to-one but not onto.

C. f is both one-to-one and onto.

D. f is neither one-to-one nor onto.

[MCQ - 1 Mark]

Correct Answer: A

8. Find the third-degree Taylor polynomial for $f(x) = e^{2x}$ centered at $x = 0$.

A. $1 + 2x + 2x^2 + \frac{4x^3}{3}$

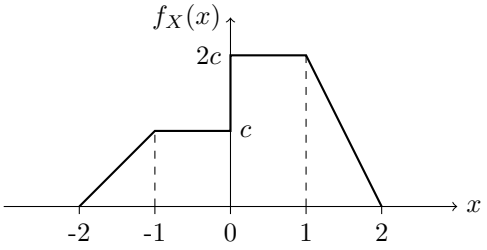
B. $1 + 2x + 2x^2 + \frac{8x^3}{3}$

C. $1 + 2x + 4x^2 + \frac{8x^3}{3}$

D. $1 + 2x + 4x^2 + \frac{4x^3}{3}$

[MCQ - 1 mark]
Correct Answer: A

9. Shown below is $f_X(x)$, the pdf of a continuous random variable X. Find the value of the constant c.



A. $\frac{2}{9}$
B. $\frac{1}{4}$
C. $\frac{1}{5}$
D. Insufficient information.

[MCQ - 1 mark]
Correct answer: A

10. A civil engineer wants to establish that the average time T to construct a new two storey building is less than 6 months. Formulate the null hypothesis H_0 and alternate hypothesis H_1.

A. $H_0 : T = 6$, $H_1 : T < 6$
B. $H_0 : T < 6$, $H_1 : T \geq 6$
C. $H_0 : T \neq 6$, $H_1 : T = 6$
D. $H_0 : T = 6$, $H_1 : T \neq 6$

[MCQ - 1 mark]
Correct answer: A

11. X is a random variable with probability density function $f_X(x) = 5e^{-a|x-2|}, -\infty < x < \infty$. The value of a is _____.

A. 1
B. 2
C. 5
D. 10

[MCQ - 1 mark]

Correct Answer: D

12. Which of the following system of equations has more than one solution?

A. $x + y = 5, 2x + 2y = 10$
B. $x - y = 4, 2x - 2y = 8$
C. $x + y = 3, x + y = 4$
D. $3x - y = 7, x + 2y = 4$

[MSQ - 1 mark]
Correct Answer: A, B

13. For two matrices $X \in \mathbb{R}^{n \times n}$ and $Y \in \mathbb{R}^{n \times n}$ which of the following are true for any X and Y.

    A. $X + Y = Y + X$
    B. $XY = YX$
    C. $X^T = X$
    D. $\alpha(X + Y) = \alpha X + \alpha Y$, $\alpha \in \mathbb{R}$

    [MSQ - 1 mark]
    Correct Answer: A, D

14. Which of the following sets form a basis for $\mathbb{R}^2$?

    A. $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$
    B. $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \end{bmatrix} \right\}$
    C. $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$
    D. $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \end{bmatrix} \right\}$

    [MSQ - 1 Marks]

    Correct Answer: A and C

15. Suppose X is a 2×2 matrix over real numbers with eigenvalues i and $-i$. Then, X can be

    A. orthogonal,
    B. symmetric,
    C. skew symmetric,
    D. invertible.

    [MSQ - 1 Mark]

    Correct Answer: A, C, and D

16. Let $Px = 0$ be a system of linear equations where P is an $m \times n$ matrix. Which of the following statements is/are true?

    A. If $m > n$, then it has no solution.
    B. If $m = n$, then it has a unique solution.
    C. If $m < n$, then it has multiple solutions.
    D. It always has a solution.

    [MSQ - 1 mark]

    Correct answer: C, D

17. Consider a function $f(x) = px + q$ defined on a closed interval $[a, b]$. Which of the following statements are true?

A. When $p > 0$, the maximum occurs at a

B. When $p > 0$, the maximum occurs at b

C. When $p < 0$, the maximum occurs at a

D. When $p < 0$, the maximum occurs at b

[MSQ - 1 mark]
Correct Answer: B, C

18. A z-test can be used for the following situation(s). Select all that apply.

A. Test the mean of a population when the population variance is known.

B. Test the variance of a population when the population variance is unknown.

C. Compare the means of two populations when the population variances are known.

D. Compare the variances of two populations when the population variances are unknown.

[MSQ - 2 mark]
Correct answer: A, C

19. Probability density functions of two random variables X and Y are given by
$f_X(x) = \begin{cases} 1 - |x| & \text{if } |x| \leq 1 \\ 0 & \text{else.} \end{cases}$
$f_Y(y) = \begin{cases} 1/2 & \text{if } 0 \leq y \leq 2 \\ 0 & \text{else.} \end{cases}$
Select all the statements which are surely true.

A. $P(X = 4) = P(Y = 1)$.

B. $P(X < 1) = P(Y < 3)$.

C. $P(X < 0, Y < 1) = 1/4$.

D. $P(X > 1) = 1/2$.

[MSQ - 1 mark]

Correct options: A, B

20. With two non-zero vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ of size $N \times 1$, we get a matrix $\boldsymbol{A} = \boldsymbol{x}\boldsymbol{y}^T$ where $(\cdot)^T$ denotes transpose. Rank of $\boldsymbol{A}$ is _____.

[NAT - 1 Marks]

Correct answer: 1.

21. The number of real 2×2 matrices which satisfy the equation $\boldsymbol{P}^2 = \boldsymbol{I}$ (Here, $\boldsymbol{I}$ denotes identity matrix) is _____.

A. 1

B. 2

C. 4

D. ∞

[MCQ - 1 Marks]

Correct answer: D

22. Consider the vectors $\boldsymbol{x} = \alpha \begin{bmatrix} 4 \\ -3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\boldsymbol{y} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$. The value of α for which $\boldsymbol{x}$ and $\boldsymbol{y}$ are orthogonal is _____. (up to 2 decimal places)
[NAT - 1 Mark]
Answer Range: -0.05 - -0.07

23. Consider the linear system of equations below.

$$\begin{aligned} x + \alpha y &= -2 \\ 2\alpha x + 18y &= 12 \end{aligned}$$

The value of α for which the system has infinitely many solutions is _____.
[NAT - 1 Mark]
Answer Range: -3 - -3

24. The dimension of the vector space of all symmetric 2×2 real matrices with trace equal to zero is _____.
[NAT - 1 Mark]

Answer: 2

25. Consider the piecewise function:

$$f(x) = \begin{cases} 3x + 1, & \text{if } x < 2 \\ ax^2 + 3, & \text{if } x \geq 2 \end{cases}$$

Determine the value of a that ensures $f(x)$ is continuous at $x = 2$. Provide your answer as a numerical value rounded to two decimal places. _____.
[NAT - 1 mark]
Correct Answer: 1

26. $\lim_{x \to 0} \left(\frac{sinx}{x} \right)^{\frac{1}{x^2}}$ =_____. (Round off up to 2 decimals)
[NAT -1 Mark]

Ans: $e^{\frac{-1}{6}}$. **Correct Answer**: 0.8 - 0.9

2 Marks

27. If boys and girls are born equally likely, then in a family of 3 children what is the probability that there are exactly 2 boys? _____. (Round off up to 2 decimals)
[NAT - 1 mark]
Correct answer: 0.37–0.38

28. X is a continuous random variable with uniform distribution in the interval $[0, 2]$. With $Y = 2X$, the joint probability $P(X < 1, Y > 1)$ is _____.
[NAT - 1 mark]
Correct option: 0.25, range 0.24-0.26

29. Let $v_1 = (0, 2)$, $v_2 = (3, -3)$, and $v_3 = (6, -2)$. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that

$$T(v_1) = w_1 \quad \text{and} \quad T(v_2) = w_2.$$

Then the value of $T(v_3)$ is.

A. $2w_1 + 2w_2$
B. $2w_1 - 2w_1$
C. $w_1 - w_2$
D. $w_2 - w_1$

[MCQ - 2 Mark]

Ans: A.

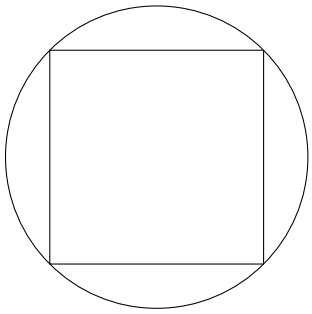
30. What is the **general solution** to the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}?$$

A. $y = C_1e^{2x} + C_2xe^{2x}$
B. $y = (C_1 + C_2x)e^{2x} + x^2e^{2x}$
C. $y = (C_1 + C_2x)e^{2x} + xe^x$
D. $y = (C_1 + C_2x)e^{2x} + x^2$

[MCQ - 2 marks]
Ans: A

31. A point is chosen uniformly at random in a circle of radius r. Inscribed in the circle is a square as shown in the figure. Find the probability that the point falls outside the square.



A. $\frac{\pi-2}{\pi}$
B. $\frac{2}{\pi}$
C. $\frac{1}{\pi}$
D. $\frac{\pi-1}{\pi}$

[MCQ - 2 mark]
Correct answer: A

32. Let S_1 and S_2 are two sets of vectors in $\mathbb{R}^n$. Suppose the vectors in S_1 are linearly independent while the vectors in S_2 are linearly dependent. Which of the following statements is/are true?

A. $S_1 \cup S_2$ is linearly dependent.

B. $S_1 \cap S_2$ is linearly independent.

C. $S_1 \cap S_2$ is linearly dependent.

D. $S_1 \cup S_2$ is linearly independent.

[MSQ - 2 mark]

Correct answer: A, B

33. Let $\boldsymbol{X}$ be a 2×2 matrix with eigenvalues 3 and -4 with corresponding eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Select all statements which are true.

A. $\boldsymbol{X}$ is not a symmetric matrix

B. $\boldsymbol{X}$ is a symmetric matrix

C. determinant of $\boldsymbol{X}$ is 24.

D. determinant of $\boldsymbol{X}$ is -12.

[MSQ - 2 Marks]
Correct options: B and D

34. A random experiment has sample space Ω. Suppose E, F are two events where $F \subset E$. Then which of the following statements are true?

A. $P(E|\Omega) = P(E)$

B. $P(E|\Omega) = 0$

C. $P(E|F) = 1$

D. $P(E|F) = 0$

[MSQ - 2 mark]
Correct options: A, C

35. $\boldsymbol{A}$ is a 2×2 matrix with eigen values -2 and 3. The smallest eigenvalue of the matrix $\boldsymbol{C} = \boldsymbol{A}^{-1} - \boldsymbol{A}$ is _____. (up to 2 decimal places)
[NAT - 2 Marks]
Answer Range -2.65 - -2.68

36. Suppose the determinant of X was 3. What is the determinant of Y? _____

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad Y = \begin{bmatrix} 2a & 2b \\ 3c & 3d \end{bmatrix}$$

[NAT - 1 mark]

37. A chip company has two factories A and B, which produce 20% and 80% of their chips. 15% of the chips produced by factory A is defective and 10% of the chips produced by factory B is defective. One of the chips made by the company is found to be defective. The probability that that particular chip was made in factory A is _____. (up to 2 decimal places)
[NAT - 2 marks]

Answer Range 0.26-0.28

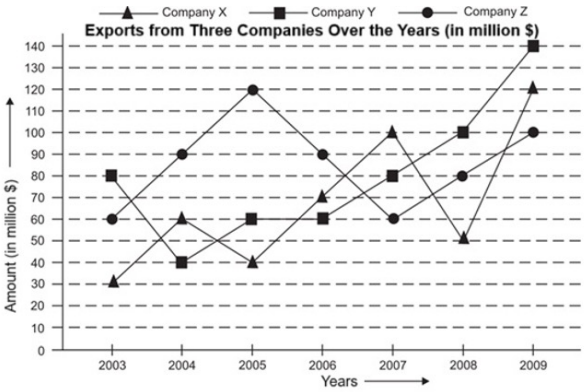
38. A rectangular field is to be enclosed using 240 meters of fencing. One side of the field borders a river and does not require fencing. What is the maximum area that can be enclosed? Provide your answer in square meters. _____.
[**NAT - 2 mark**] Ans: 7200

39. A fair die with numbers $\{-1, 0, 1, 2, 3, 4\}$ is thrown twice. If the sum of the dice is 4, find the probability that the first die shows 2? _____. (Round off up to 2 decimals)
[**NAT - 2 mark**]
Correct answer: 0.13–0.14 (exact answer 5/36)

General Aptitude

All MSQs, 1 mark each **Common data for Questions 40-41**

The annual export amount of three companies over the years 2003-2009 are shown in a line graph. Study the following line graph and answer the questions given below:



40. Average annual exports during the given period for Company Y is approximately what percent of the average annual exports for Company Z?

(a) 93.33
(b) 87.12
(c) 91.12
(d) 95.15

Correct Option: A

41. In how many of the given years, were the exports from Company Y more than the average annual exports over the given years?

(a) 2
(b) 3
(c) 5
(d) 6

Correct Option: A

42. The number of coins of $ 1, $ 5, and $ 10 denominations that a person has are in the ratio 5:3:13. Of the total amount, the percentage of money in $ 5 is

(a) 21%

(b) 14%

(c) 10%

(d) 30%

Correct Option: C

43. A water tank is filled in 6 hours by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the water tank?

(a) 20 hrs

(b) 25 hrs

(c) 42 hrs

(d) 44 hrs

Correct Option: C

44. A mother is 24 years older than her daughter. In 6 years, the mother,Äôs age will be three times the daughter,Äôs age. How old is the mother right now?

(a) 42

(b) 30

(c) 36

(d) 24

Correct Option: B

Common Passage for Q45-Q49
From Yuval Harari's Sapiens: A Brief History of Humankind

During the first half of the twentieth century, scholars taught that every culture was complete and harmonious, possessing an unchanging essence that defined it for all time. Each human group had its own world view and system of social, legal and political arrangements that ran as smoothly as the planets going around the sun. In this view, cultures left to their own devices did not change. They just kept going at the same pace and in the same direction. Only a force applied from outside could change them. Anthropologists, historians and politicians thus referred to 'Samoan Culture' or 'Tasmanian Culture' as if the same beliefs, norms and values had characterised Samoans and Tasmanians from time immemorial.

Today, most scholars of culture have concluded that the opposite is true. Every culture has its typical beliefs, norms and values, but these are in constant flux. The culture may transform itself in response to changes in its environment or through interaction with neighbouring cultures. But cultures also undergo transitions due to their own internal dynamics. Even a completely isolated culture existing in an ecologically stable environment cannot avoid change. Unlike the laws of physics, which are free of inconsistencies, every man-made order is packed with internal contradictions. Cultures are constantly trying to reconcile these contradictions, and this process fuels change.

45. How would you characterise the scholars's point of view on culture in the first half of the twentieth century, as described in the first paragraph?

(a) Essentialist

(b) Constructionist

(c) Primitive

(d) Relative

Correct Option: A

46. According to the author, which is not a true statement about contemporary understanding of culture?

(a) Cultures try to reconcile internal contradictions.

(b) Cultures are constantly mutating.

(c) Cultures are attentive to internal and external processes of change.

(d) Culture needs to be understood and studied according to scientific principles.

Correct Option: D

47. Which of the following means the same as Flux? (1 mark)

(a) Exodus

(b) Convergence

(c) Fluidity

(d) Emigration

Correct Option: C

48. Select the one word that has most likely the same meaning as the italicized word in the context of the sentence.
The back-benchers mostly feel *alienated* from the rest of the class.

(a) Happy

(b) Joyous

(c) Angry

(d) Estranged

Correct Option: D

49. Choose the word when substituted for the blank space, best completes the meaning of the sentence.
The President was not known for being obstinate; on the contrary, her cabinet members praised her _____

(a) Rigidity

(b) Experience

(c) Meticulousness

(d) Flexibility

Correct Option: D